Example
The function defined by \( y = \sqrt{r^2 - x^2} \) has as its graph a semicircle of radius \( r \) with center at \((0,0)\).
Find the volume that results when each semicircle is rotated about the \( x \)-axis.
\[
f(x) = \sqrt{r^2 - x^2}
\]
\[
V = \int_{-r}^{r} \pi f(x)^2 \, dx
\]
\[
= \int_{-r}^{r} \frac{\pi}{3} (r^2 - x^2) \, dx
\]
\[
= \pi \left( r^3 - \frac{x^3}{3} \right) \bigg|_{-r}^{r} = \frac{4}{3} \pi r^3
\]

Example
Find a volume of an ellipsoid
\[
f(x) = \frac{b}{a} \sqrt{a^2 - x^2}
\]
\[
V = \int_{-a}^{a} \pi [f(x)]^2 \, dx
\]
\[
= \int_{-a}^{a} \frac{\pi}{a^2} \frac{b^2}{a^2} (a^2 - x^2) \, dx
\]
\[
= \pi \left( b^2 x - \frac{b^2 x^3}{3a^2} \right) \bigg|_{-a}^{a} = \frac{4}{3} \pi ab^2
\]